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REVIEW OF A DOCTORAL DISSERTATION

Augmented Lagrangian-Based Algorithms for Separable Non-convex Optimization with Applications in Network Routing and Machine Learning

by Anthony Chukwuemeka Nwachukwu, M.Sc.

prepared for the

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1. General overview

The dissertation addresses a research problem of high contemporary relevance, situated at the intersection of large-scale optimization, distributed computing, and applications in network routing and machine learning. The growing scale and heterogeneity of modern computational systems makes classical centralized optimization approaches increasingly impractical. In this context, the focus on distributed and, in particular, asynchronous optimization methods responds directly to the real-world challenges such as communication delays, processor heterogeneity, and scalability constraints. By targeting separable non-convex optimization problems, the dissertation tackles a class of problems that are theoretically challenging as well as practically important. The dissertation has both a scientific and applied character, proposing a solution to important and difficult optimization and IT issues.

2. The concept of the dissertation and its implementation

The dissertation, with a total of 149 pages, is written in English with a two-page summaries in Polish and English. It consists of nine chapters and a short, four-page introduction, which initially formulates the goal of the work, describes the research problem, the key contributions and the hypothesis of the dissertation. In the last part of dissertation, the author discusses the contributions summary, the limitations of the proposed solutions as well the possible directions for further work. The attached bibliography consists of 96 items, primarily specialist journals and international conference proceedings. It is a well-curated list of relevant works related to the subject of the dissertation. The proposed bibliography is complete and confirms indirectly the fact that the doctoral student has a very good knowledge in the field of technical information technology and telecommunications.

Chapter 2 provides an introduction to the theoretical foundations covering optimization problems such as convex analysis, optimality conditions, Lagrangian duality and optimality (including the Karush-Kuhn-Tucker (KKT) condition and the augmented Lagrangian), continuity, Lipschitz conditions and stability, fixed-point iterations and convergence, as well as monotone operators and variational properties. Chapter 3 is dedicated to a review of classical decomposition methods, including the ordinary Lagrangian relaxation method, augmented Lagrangian method, Bertsekas decomposition method, Tanikawa-Mukai decomposition method, Tatjewski's decomposition method, and the SALA decomposition algorithm in the ADMM version. In Chapter 4, the convergence analysis of the asynchronous Bertsekas augmented Lagrangian method is discussed. A proof of the convergence theorem for the asynchronous algorithm is presented along with useful lemmas.

In Chapter 5, the author considers an especially important issue, as the network optimization problem of simultaneous routing and bandwidth allocation in energy-aware networks is presented. Here, selected implemented methods are tested on the network problems of different sizes. The tested networks consist of loosely connected node clusters with strongly connected nodes. The experimental results presented in summary tables and figures illustrate well the studied optimization algorithms. In Chapter 6, the implemented two variants of the Bertsekas methods (the synchronous and the asynchronous) are investigated. Using the same dataset as in the Chapter 5, numerical experiments were conducted with the use of Python. The experimental results are given for small, large and extra-large problem objective values (routing). The short Chapter 7 presents the solution of regularized linear systems with augmented Lagrangian algorithms. In numerical experiments, three linear systems, designed to reflect varying data structures and matrix conditions, are implemented. Among them are: an image inpainting task, using randomized diagonal measurement operators, and a biomedical dataset. Chapter 8 explores both synchronous and asynchronous update schemes within the Bertsekas augmented Lagrangian framework, highlighting their algorithmic structures and practical performance across diverse datasets. In the two last chapters (9 and 10), the author presents the distributed formulation of the constrained K-Means problem and investigates the impact of asynchrony on K-Means clustering through empirical comparisons on large-scale datasets. A summary of contributions and suggestions for future research are presented in the Chapter 11.

On the basis of a brief overview of the content of the dissertation, I conclude that Anthony Chukwuemeka Nwachukwu, M.Sc., has demonstrated a high degree of ability to formulate scientific problems, depending on the analysis of the current state of knowledge available in the contemporary international literature. The research question: *„Which Lagrangian-based optimization algorithms are most effective for solving large-scale, distributed non-convex problems, and how can we design and rigorously analyze asynchronous versions of these methods?* has been solved using appropriate methods and algorithms of the non-convex optimization theory. Extensive experimental studies were carried out using both available synthetic benchmarks and real-world datasets. The thesis is of a theoretical nature and the description of the problem and its solution uses the mathematical formalism appropriate for the discipline of technical informatics and telecommunications.

3. Original achievements and the author's contribution

By answering the research question and achieving the formulated goals, the author obtained several interesting and original scientific results confirmed both by formal proof and by experimental computer research, using available synthetic benchmarks and real-world datasets. The important scientific achievements of Anthony Chukwuemeka Nwachukwu, M.Sc., and his main contribution to the discipline of *Technical informatics and telecommunications* include, among others, the following:

a) The major achievement of the dissertation is the development of an original asynchronous variant of the Bertsekas augmented Lagrangian decomposition algorithm. While the Bertsekas method is a well-established and powerful approach in convex and certain non-convex settings, its extension to an asynchronous computational model is nontrivial. The author successfully reformulates the algorithmic framework in order to allow local computational nodes to update their variables without resorting to global synchronization and with no risk of using potentially outdated information. This constitutes a genuine methodological contribution that goes beyond straightforward adaptations of existing synchronous algorithms.

b) A special significance of the work lies in the rigorous theoretical analysis accompanying the proposed algorithm. The dissertation provides a detailed convergence analysis under a bounded-delay asynchronous model, which is well aligned with standard assumptions used in the analysis of asynchronous iterative methods. Under appropriate regularity conditions, including smoothness, Lipschitz continuity of gradients, and suitable constraint qualifications, the author proves convergence of the proposed method to Karush–Kuhn–Tucker points of the original non-convex problem. Given the inherent difficulty of establishing convergence guarantees for asynchronous algorithms in non-convex settings, this analysis represents a substantial theoretical contribution.

c) The dissertation is distinguished by its comprehensive and well-organized review of augmented Lagrangian–based decomposition methods. Classical dual decomposition, the method of multipliers, ADMM, the Bertsekas method, Tatjewski's approach, and the SALA algorithm are presented in a systematic manner. Importantly, this review is not merely descriptive; it serves as a foundation for a comparative empirical study that validates the choice of the Bertsekas method as the algorithmic core of the dissertation. This critical and selective use of literature demonstrates the author's strong understanding of the field and the ability to position their work within existing research.

d) The breadth and depth of the numerical experiments constitutes another achievement. The proposed algorithms are validated on a wide range of problems, including simultaneous routing and bandwidth allocation in energy-aware networks, the solution of large regularized systems of linear equations, and distributed K-means clustering. These applications cover both network optimization and machine learning, thereby illustrating the universality of the proposed framework. The experiments are performed on both synthetic benchmarks and real-world datasets, enhancing the credibility and practical relevance of the results.

e) The dissertation demonstrates that the asynchronous Bertsekas algorithm achieves objective values comparable to those obtained by synchronous implementations, while significantly reducing wall-clock computation time as the degree of parallelism increases. This empirical evidence strongly supports the main claim of the dissertation, namely that asynchronous augmented Lagrangian methods can exploit modern parallel computing environments more efficiently than their synchronous counterparts.

f) The work also succeeds in bridging the gap between theory and practice. The mathematical analysis is closely connected to the algorithmic design, and the assumptions made in the theoretical part are reflected in the implementation and experiments. This coherence between theoretical development and practical validation is a notable strength of the dissertation.

The original scientific results presented in the thesis and the contribution of Anthony Chukwuemeka Nwachukwu, M.Sc., to the discipline of *Technical informatics and telecommunications*, confirmed to a large extent by theoretical and experimental research, in some part have already been published in several collaborative scientific works, namely in the following journlas: *International Journal of Electronics and Telecommunications* (2025, 70 points), *Energies* (2024,140 points) and *Task Quarterly* (2020, 20 points).

Finally, the dissertation is well structured and clearly written. The progression from theoretical foundations, through a review of existing methods, to the development of a new algorithm and its applications is logical and easy to follow. The figures, tables, and numerical results are generally well presented, and the conclusions are supported by the given evidence. Overall, the dissertation demonstrates the author's maturity as a researcher and their ability to conduct independent, high-quality scientific work.

4. Remarks and comments

Overall, I rate the entire dissertation positively in terms of content and editing. Some of the comments below are of substantive and some of editorial nature:

a) One of the main issues regards the assumptions underlying the theoretical convergence analysis:

- While the assumptions of smoothness, Lipschitz continuity of gradients, and appropriate constraint qualifications are a standard in the literature, they are relatively strong and may not always be satisfied in practical large-scale applications. The dissertation could have benefited from a more extensive discussion of how restrictive these assumptions are in practice and to what extent is the proposed method inclined to their violation.

- The convergence results establish convergence to KKT points, which is the strongest result one can typically expect in a non-convex optimization. However, the practical implications of convergence to stationary points, as opposed to global optima, are discussed nonexhaustively. A more thorough discussion of the quality of the obtained solutions in non-convex settings, supported, preferably, by additional empirical observations, would strengthen the practical interpretation of the theoretical results.

b) Although the dissertation includes comparisons with several classical and well-established methods, such as ADMM and other augmented Lagrangian–based approaches, the set of benchmark algorithms could be expanded. In particular, recent developments in asynchronous non-convex optimization, including novel variants of asynchronous ADMM or stochastic methods, are considered not extensively. Inclusion of such methods could provide a broader perspective on the relative performance of the proposed algorithm.

c) Editorial remarks:

- The theoretical background chapter, while thorough and mathematically sound, is relatively extensive. Some of the material presented is well known to experts in optimization and could have been condensed. A more concise presentation of the standard results might have allowed for greater emphasis to be put on the novel theoretical contributions, specific to the asynchronous algorithm developed in the dissertation.

- From a visual perspective, the numerical results section includes a very large number of figures and tables. While this reflects the breadth of the experimental study, it can make it difficult at times for the reader to identify quickly the most important trends and conclusions. The readability of those sections could be improved by more frequent use of summary tables or explicit statements highlighting the key empirical findings.

- Although the dissertation includes a section on limitations and future work, the limitations of the proposed method could have been more explicitly emphasized throughout the text. Issues such as sensitivity to parameter selection, the impact of very large communication delays or scalability limits in extremely large or highly heterogeneous systems are mentioned only briefly. A more critical discussion of these aspects would provide a more balanced assessment of the applicability of the method.

- In doctoral dissertations, it is recommended to include summaries of individual chapters, in which the author introduces new content and results. The lack of such summaries somewhat hinders the substantive and logical conclusions that could be drawn between chapters.

4. Summary

In conclusion, the identified weaknesses do not undermine the overall quality or significance of the dissertation. Rather, they point to natural directions for further research and refinement of the theses. The dissertation remains a strong and valuable contribution to the field of distributed optimization.

Considering all aspects of the doctoral dissertation by Anthony Chukwuemeka Nwachukwu, M.Sc., including the noted scientific and applied achievements in particular, as well as the list of co-authored publications, and taking into account the substantive and formal discussion comments, that do not have a significant impact on the final assessment of the dissertation:

a) I declare that the doctoral dissertation meets all the requirements of the Law on Higher Education and Science of 20 July 2018 (Article 168) in the field of engineering and technology in the discipline of *Technical informatics and telecommunications;*
b) I request that the dissertation be accepted by the *Discipline Council for Technical Informatics and Telecommunications* of Warsaw University of Technology.